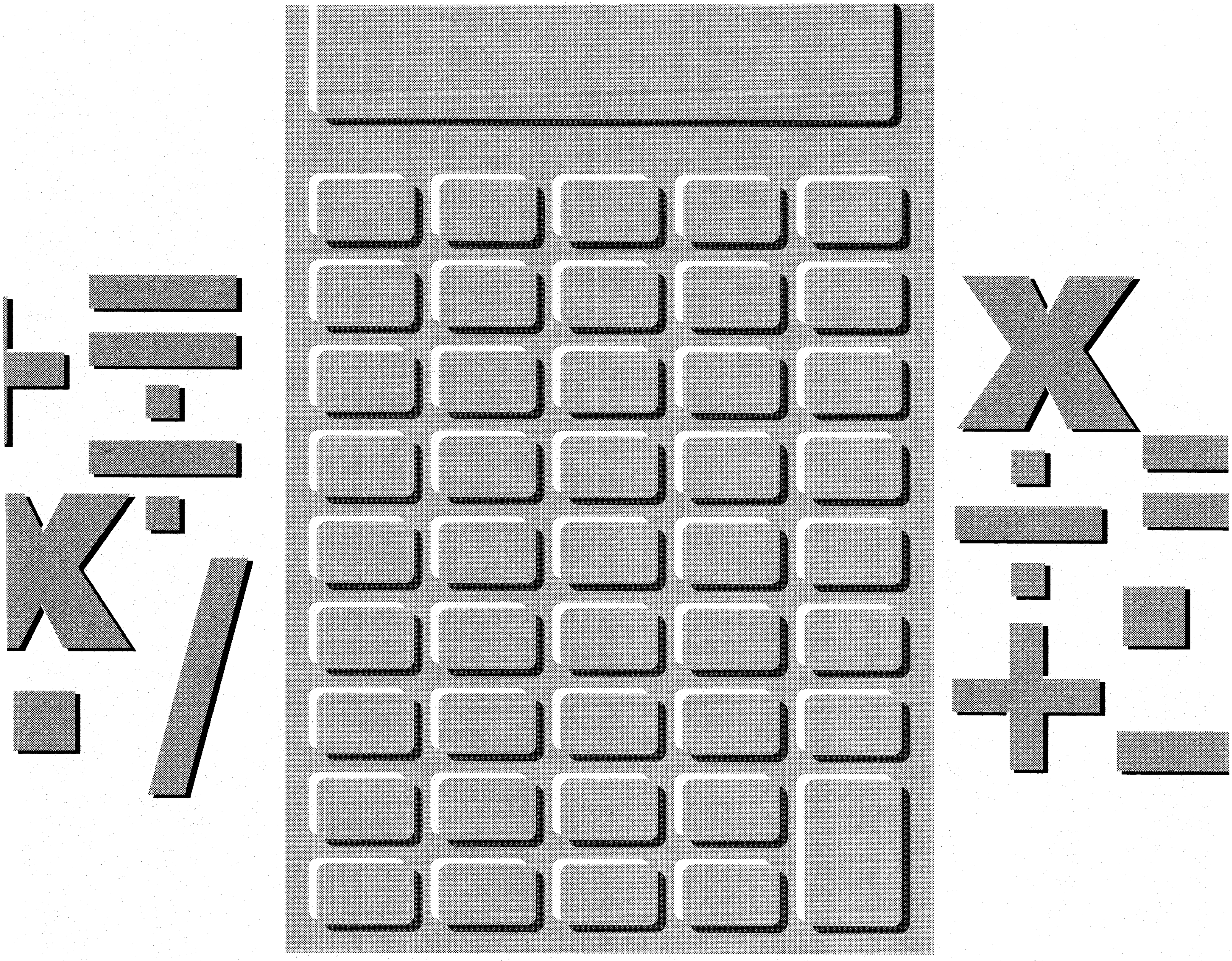

GUIDE FOR RATING REGENTS EXAMINATIONS IN MATHEMATICS

1996 EDITION

NEW YORK
THE STATE OF LEARNING

The University of the State of New York
The State Education Department
Albany, New York 12234

Guide for Rating

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in Mathematics

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Introduction

The purpose of this guide is to provide a set of directions, together with some examples, to assist teachers in the rating of student responses for Regents examinations in sequential mathematics, course I, II, and III. While it is not possible to anticipate all the possible questions that may arise, the suggestions and examples in this guide will deal with those that tend to occur most frequently.

In all work in mathematics the aim should be accuracy—not only in the mechanical aspects but also in the aspects that require reasoning, judgment, and application. In every instance where an error occurs, to make proper deductions, a careful consideration should be given to the relative importance of the error.

The principal is responsible for establishing rating procedures that will assure reasonable confidence in the accuracy of the scores assigned to the Regents answer papers by individual teachers or by committees of teachers. The criterion in all cases shall be that the rating assigned to a student's paper shall be a fair and accurate rating of that paper.

The scoring key provided by the Department is the official key and teachers must rate according to this key. Unless otherwise specified, mathematically equivalent answers are allowed. It should be understood that any special ruling or rating as indicated on the Regents examination or scoring key takes precedence over any suggestion made in this guide and is not considered as conflicting. The staff of the State Education Department is available to answer questions concerning the rating of Regents examination papers. Call or write Lynn Richbart, New York State Education Department, Albany, New York 12234 (518: 474-3935).

The following rating suggestions are divided into seven sections: general (GEN), algebra (ALG), geometry (GEOM), graphing and coordinate geometry (GRAPH), circular functions (CIR FN), logic (LOGIC), and probability and statistics (PROB–STAT). These sections are followed by typical Part I, II, and III examination questions and responses that have been rated to show the appropriate credit allowed. Each rated question has an explanation of that rating. Many of the rating guidelines are referenced by number to questions in the example section. Each question is, in turn, referenced to one or more of the rating guidelines using GEN, ALG, GEOM, GRAPH, CIR FN, LOGIC, or PROB–STAT.

Directions and Suggestions for Administering Regents Examinations in Mathematics

DIRECTIONS TO TEACHERS

1. All students must have scientific calculators available. Most nonprogrammable and nongraphing scientific calculators are permissible. Calculators that can communicate with other calculators through infrared sensors are **not** permitted. Since the State will not be including the logarithmic and trigonometric tables in the examination booklets and such tables will **not** be permitted in the examination room, the scientific calculators **must** have these features. Calculators having other capabilities such as fractional notation, factorials, combinations, permutation evaluations, π, or various statistical features are also permitted. Instructional cards or calculator operations manuals are **not** permitted during the examination.
2. All students must have available a straightedge, and when necessary, a compass.
3. All students must be provided with paper on which they will write their answers to Part II (and Part III for sequential math, course II).
4. All students must be provided with graph paper. Coordinate paper with prelabled axes or scales must not be used.

DIRECTIONS TO STUDENTS

Before a Regents examination begins, the students should be advised:

1. To remove all books, notes, or other aids from their reach or sight during the examination
2. To read the questions carefully and to follow instructions
3. To make sure that they have completely filled in the heading of the answer sheet and answer booklet
4. To use a pen in writing their answers, except for making drawings and diagrams or marking machine-scorable answer sheets
5. Not to use red ink or red pencil
6. Not to erase answers written in ink, but to cross out the original answer with a single line and then write the new answer
7. To sign the pupil declaration at the proper time
8. That any attempt either to obtain or to give aid will result in the termination of their examinations
9. That no partial credit will be allowed for Part I questions; that answers are to be written in the spaces provided on the separate answer sheet (Where applicable, answers may be left in terms of π or in radical form.)
10. That they must clearly indicate the necessary steps—including appropriate formula substitutions, diagrams, graphs, and charts—for Part II (or III) questions (Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown.)

IMPLICATIONS FOR ADMINISTRATION

1. Students must use blue or black pen (not red) to write their answers. The only exception is that pencil may be used for graphs/diagrams and their labels.
2. Neither teachers nor students may use correction fluid.
3. When any part of a question is answered on graph paper, the entire question should be answered on graph paper if at all possible.
4. Examination papers submitted to the Department for review should include only answer papers that have been scored. Do not include scrap paper, test booklets, etc.

Suggestions for Rating

GENERAL

1. Only red pencil or red ink may be used for rating Regents papers.
2. A rater may **not** correct a student's work by making insertions or changes of any kind.
3. Raters should indicate student errors with a checkmark (✓) and indicate the credit allowed for each part of a question. Do not use a checkmark (or any other symbol) to indicate a correct answer.
4. Committee rating of examinations is a technique in which each member of the committee rates only one or two Part II or Part III questions. This "division of labor" has been shown to be both effective and efficient since each rater has fewer questions and issues to consider.
5. If an entire examination is to be rated by one person, the rater must first work through the entire examination and construct his/her own answer key. This answer key should then be compared with the official scoring key. This process will give the rater a valuable "feel" for the examination, indicate any potential trouble-spots before the rating begins, and identify questions which may have several forms of acceptable (equivalent) answers.
6. If a student answers more questions than required on any part of an examination where choice is allowed, the rater shall score only the required number of questions in numerical order by item number. For example, if a student does questions 42, 37, 39, 36, and 40 (in this order), when only four were required, then only questions 36, 37, 39, and 40 are to be rated. The rater should include those questions answered on separate paper, e.g., graphs, when determining numerical order.
7. Answers for those parts of the examination for which answer sheets are not provided by the Department should be written on paper (ruled or unruled) provided by the school. In unusual circumstances, a student's answer appearing on scrap paper may be rated if the student's name appears on the scrap paper *and* if the answer has *not* been copied onto the school-provided answer sheet.
8. Coordinate paper with prelabeled axes or scales must not be used.
9. In scoring Part II and Part III, the total cumulative score for *that part* should be entered in red in the margin to the right of each question. Totals for each part of an examination are to be entered in the proper spaces on the Part I answer sheet.
10. Any equivalent form will be accepted as a correct answer, including rational approximations of π or square roots to at least three significant figures, unless a specific form is indicated in the question. [Ex–2(ii),(iii); Ex–25*a*]
11. Deductions on a Part II or Part III question must be integral; that is, fractional credits are **not** allowed. No partial credit is allowed on Part I.
12. Answers to latter parts of a question which are based on former parts must be graded carefully if a mistake has already been made. A seemingly incorrect response may receive full credit if it is based on a previously made mistake. The rationale is that students should not be penalized more than once for the same mistake. [Ex–12*e*; Ex–16; Ex–18*b*; Ex–19*b*; Ex–24*b*,*c*,*d*,*e*; Ex–26; Ex–28; Ex–29*c*; Ex–32; Ex–33*b*]
13. On Part II and Part III, students should show all work. Students should clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, and charts. However, credit should not be deducted in those instances where calculations

can be performed by mental arithmetic or a scientific calculator. [Ex–21(ii); Ex–27; Ex–30(i); Ex–38*b*]

14. When deducting credits, the rater must be careful to determine whether the error is arithmetic or due to a violation of some principle. An arithmetic error which does not change the nature of the question should receive a deduction of 1 point, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 to 50 percent, depending on the relative importance of the principle in the solution of the problem. The maximum deduction for arithmetic errors in each part of a question is 1 point. [Ex–21(iii); Ex–23; Ex–24*a*; Ex–25*b*; Ex–26; Ex–28; Ex–30(iii); Ex–31]
15. If a Part II or Part III question has deductions totaling more than 50 percent of the allotted credit, then the rater may switch from deducting for mistakes to adding for correct information given. Under such circumstances, the final credit assigned would rarely exceed 40 percent of the total allotted credit. [Ex–20]
16. Partial credit may be given for incomplete solutions of problems or exercises when it is clear that the method used could be carried on to a correct conclusion. In such cases, however, the work should be carried far enough to indicate clearly that the student is certain of the procedure and that failure to finish the problem is due either to lack of time or to the inability to see the next step in the solution. The amount of credit allowed depends upon the amount and degree of importance or difficulty of the work that has been done correctly. No credit should be given for merely enumerating extraneous facts or principles. [Ex–29*a*,*b*]
17. In the solution of numerical exercises, not more than 20 percent credit is allowed for writing the correct formula(s). If the formulas are given, then no credit should be allowed.
18. No partial credit is allowed for solving an incorrect equation which is considerably easier than the intended one; however, partial credit should be granted for solving incorrect equations of equal difficulty. *For that purpose, these alternative solutions should be carefully checked.* [Ex–28; Ex–31]
19. Answers to questions involving computation should not be given full credit unless the degree of accuracy specified in the question is observed. Computations should be performed with appropriate accuracy (at least one decimal place more than the required answer), with results rounded as required. Students should be instructed to take full advantage of the calculator by using all of the digits produced by the calculator during computation. Rounding to the desired degree of accuracy should be done only at the end of the computation. Failure to do so should receive a deduction of one credit. [Ex–36]
20. Numerical illustrations are not to be accepted in lieu of a proof. In general, proofs involving special cases should receive little or no credit, never more than 50 percent.
21. In the solution of equations, the solving of verbal problems, the demonstration of original geometric exercises, the proofs of trigonometric identities, etc., little credit, if any, should be deducted for the omission of those steps which, in the judgment of the examiner, do not detract from the merit of the work. [Ex–14(iii); Ex–21(ii); Ex–22]
22. When a student's method of solution differs from that which is generally accepted, but is mathematically sound and employs acceptable techniques, no deduction should be made. [Ex–37; Ex–41(ii)*b*]
23. The presence of irrelevant statements in answers to Part II or Part III questions is a continuing dilemma for raters. Correct irrelevant statements that do not seriously detract from the solution as a whole should receive no deduction, whereas incorrect irrelevant statements should not be penalized more than 20 percent. [Ex–12*d*; Ex–22; Ex–30(iii)]
24. In problems involving computation by means of logarithms, a premium should be placed on the theory.
25. When a question requires solving equations graphically and estimating the answer to the

nearest tenth, the allotted credit for the reading should be granted if it is estimated to within one tenth from the graph as made.

26. If the question does not specify how a verbal problem is to be solved, either algebraic or graphic methods are acceptable. In addition, any method such as "make a table or chart" or "trial and error" may be used. If a student solves the problem algebraically or graphically, the work should be rated in accordance with this rating guide. If a student chooses to solve the problem by a method other than algebraic or graphic, full credit should be given only if the student shows or explains how he/she arrived at the answer. [Ex–40(i),(ii); Ex–41(i)*b*; Ex–42]

27. To simplify calculations with the scientific calculator, values generated by the calculator for trigonometric functions, logarithms, roots, and π may be used. However, in no instance should approximations of less than three significant figures be used. [Ex–30(ii); Ex–36; Ex–37; Ex–38*a*]

ALGEBRA

1. Acceptable equivalent answers commonly occur in examples involving:
   (a) Fractions sometimes not reduced to lowest terms [Ex–1(ii)]
   (b) Expressions involving square roots:
   $\sqrt{121}$ *or* 11, $\sqrt{48}$ *or* $4\sqrt{3}$ *or* $2\sqrt{12}$
   (c) Equivalent percents, decimals, and fractions
   (d) Unless otherwise specified, rational approximations correct to at least three significant figures [Ex–2(ii),(iii)]
   (e) Equivalent absolute values
   (f) Numerical expressions where the order in which the indicated operations are performed has no effect on the result (However, simply substituting values into an equation or formula is not sufficient.) [Ex–2(i); Ex–8]
2. If the directions say to simplify, then all answers containing fractions should be single fractions in lowest terms, and those containing radicals should be in "simplest" radical form, i.e.,
   (a) No radicands with perfect square factors
   (b) No fractional radicands
   (c) No irrational denominators
3. Part I questions for which a student gives more information than is requested need to be examined carefully.
   For example:
   (a) If both values for the variables in a system of equations are correctly listed, either as a pair of equations or as an ordered pair, when only one value was asked for, full credit should be allowed. If the second value is not correct, no credit should be allowed. [Ex–4(i),(ii)]
   (b) If only the positive or negative root of an equation (or square root) is asked for, no credit should be allowed when additional roots of a different nature are listed. [Ex–3; Ex–6]
   (c) If an item asks for either the slope or y-intercept of a linear equation and *both* are correctly listed and clearly identified, full credit should be allowed; however, the use of m and b as labels is *not* to be construed as clearly identified. [Ex–5(i),(ii),(iii)]
4. In the setting up of equations to be used in the solution of verbal problems, partial credit may be given for the formation of incorrect equations where there is a minor error in reasoning.
5. For verbal problems worth [a,b], allow a credits for the analysis, which consists of the representation of the unknowns and the formation of the appropriate open sentence(s) from which the unknowns are to be obtained. The computation is worth b credits. [Ex–20; Ex–34(i),(ii)]
6. In verbal problems, the unknowns must be identified. The credit for analysis of these problems should be divided to allow no more than 20 percent for representing the unknowns, with the remaining credit for the equation(s). [Ex–20; Ex–31]

7. In the solution of a verbal problem, a properly labeled diagram or chart may be considered as representing the unknowns.
8. In the analysis of a verbal problem that could be solved by an inequality, the use of an equation is not necessarily an error in principle. [Ex–34(i),(ii)]
9. A problem for which only an algebraic solution is acceptable should receive no more than 20 percent of the total allotted credit when only an arithmetic solution is shown or when only the correct answer appears. [Ex–20]
10. For verbal problems which lead to equations with multiple roots, 1 credit should be deducted when an inappropriate root has not been clearly rejected. [Ex–25*b*]
11. If a problem calls for the algebraic solution of a pair of simultaneous equations, then only an algebraic solution should receive full credit. A correct graphic solution using coordinate axes should receive no more than 50 percent. If required, a proper check would also receive the specified credit.
12. In checking by substituting particular values, no credit should be given unless it is done in the original equations. In checking systems of equations, no credit is allowed if the check involves only substitution into the equation from which the second unknown was determined. [Ex–32]
13. For a Part II question requiring more than one answer, each part should be rated by deducting 1 credit for each incorrect or omitted answer from the total credits allotted for that part. [Ex–15]
14. The expression $\{\phi\}$ may not be accepted when the answer is the null set, i.e., $\{\ \}$ or ϕ. Phrases such as "no solution" or "none" are acceptable.
15. When a question asks for a complex number to be expressed in $a + bi$ form, answers such as $\frac{5 + 6i}{3}$ or $\frac{1}{3}(5 + 6i)$ are not acceptable.

GEOMETRY

1. In all construction problems, accurate drawings using a straightedge and compasses are required and all construction lines must be shown. If the figure to be constructed is a subset of the construction line(s), then no deduction should be made for drawing the line(s), e.g., the altitude of a triangle. Failure to label, when required to do so, should result in the proper deduction. If the description of a construction problem is given in lieu of a drawing, not more than 50 percent of the assigned credit, and usually less, may be allowed. If the construction is on Part I, however, then no partial credit may be given. [Ex–10]
2. If, instead of a direct proposition as called for on the examination paper, the student correctly proves a converse of the proposition, 50 percent of the credit may be granted *provided the proofs are analogous and of relatively equal difficulty.*
3. "Reasoning in a circle" should not necessarily invalidate a whole proof. If credit is to be given in such cases, the amount should be determined by the extent to which the work shown could be used in establishing a logical proof.
4. There is a valid concern as to which geometric principles or theorems may legitimately be used as reasons assigned in the proof of original exercises. In the proof of original exercises, students should be cautioned to restrict themselves to the use of propositions as they appear in the current course guide published by the Department and to the use of those propositions presented by their instructor. [Ex–22]
5. In general, equal credit is not assigned to each step of a geometric proof. Some steps are more important than others and the assignment of credits should reflect this. For example, [2,2,3,2,1] might be the credits assigned to a "statement-reason proof" consisting of five steps. The proof should always be rated as a whole to determine the credits for the individual steps. [Ex–14(iv); Ex–22]

6. In the case of original exercises, credit not to exceed 20 percent may be granted for drawing the figure not given and stating the hypothesis and conclusion *in terms of the figure*, provided that it is obvious that the student's work shows an understanding of the problem.
7. In geometry, proofs involving special cases should receive little or no credit. Special cases that do preserve generality should never receive more than 50 percent credit.
8. For geometric verbal problems with a credit allotment of [a,b], allow a credits for the analysis part—which consists of drawing the correct figure, the statement of formula(s) to be used, and the formation of the equation(s) from which the unknown parts are to be obtained. The computational part is worth b credits. [Ex–25b]
9. Answers that may be left in terms of π are considered equivalent to those obtained by using 3.14 or approximations of π obtained from the calculator. [Ex–9]

GRAPHING AND COORDINATE GEOMETRY

Note: See sections on circular functions and probability and statistics for specific comments about graphing in those areas.

1. Graphs must be drawn on graph paper with straightedge (ruler) and compasses, where applicable, and with reasonable accuracy.
2. The common deductions for graphing problems are:
   (a) Axes not labeled: –1
   (b) Failure to label graphs when more than one is to be drawn on the same set of axes (no deduction if only one line or curve is not labeled): –1 [Ex–33a]
   (c) Incorrect solid or dotted line: –1 [Ex–33a]
   (d) Failure to complete graph over the specified domain: –1 or –2 [Ex–23]
   (e) Failure to indicate a scale other than a unit scale: –1
   (f) Nonlinear equations graphed as a set of line segments: –1
   (g) Failure to use straightedge: –1
3. When rating the graphing of a system of linear equations with a credit allotment of [8,2], allow:
   (a) 3 credits for each correct line
   (b) 2 credits for indicating the coordinates of the correct solution (the point of intersection of the lines drawn) [Ex–39(i),(ii)]
   (c) 2 credits for the check [Ex–39(ii)]
4. When the graphing of a pair of inequalities with a credit allotment of [8] is rated, each inequality is worth 4 credits with:
   (a) 3 credits for graphing the correct solid or dotted line
   (b) 1 credit for shading the correct region [Ex–33a]
5. When rating the graphing of a system of equations consisting of one linear and one quadratic equation with a credit allotment of [8,2], allow:
   (a) 4 credits for the correct graph of the quadratic equation (minus 1 credit if the graph is drawn as a set of line segments)
   (b) 2 credits for the correct line
   (c) 2 credits for listing the coordinates of the correct solution(s) (the point(s) of intersection of the curves drawn)
   (d) 2 credits for the check
6. If a problem calls for the graphical solution of a pair of simultaneous equations, then only a graphical solution using coordinate axes should receive full credit. A correct algebraic solution should receive no more than 50 percent. If required, a proper check should receive the specified credit.
7. When the problem involves finding the area of a polygon whose vertices are given:
   (a) The correct answer with no work shown receives 20 percent credit [Ex–21(i)]
   (b) A diagram that indicates the method of solution, along with the correct solution, should receive full credit [Ex–21(ii)]
8. In coordinate geometry proofs, the mere presence of the appropriate computation is not sufficient for full credit. Conclusions should

be stated, with appropriate written reasons supplied. [Ex–17; Ex–26]

9. The coordinates of a point (a,b) may also be expressed as the pair of equations $x = a$ and $y = b$. The expressions $\{a,b\}$ and a,b are not acceptable equivalents for the ordered pair (a,b).
10. Algebraically equivalent equations of lines, circles, etc., are acceptable. [Ex–7(i),(ii)]
11. Using an incorrect pre-image when performing a transformation from a point to its image is considered a minor error as long as it is clear what point has been used as the pre-image. The principle in this case is the proper use of the transformation. [Ex–35*c*,*d*]
12. A line reflection in an incorrect line is considered to be a major error and should receive a deduction of at least 50 percent.

CIRCULAR FUNCTIONS

1. Full credit for the proof of an identity cannot be granted unless it is evident that the procedure followed and the transformations used in the solution are clearly demonstrated by the pupil. For an identity, the rater should deduct 50 percent for the use of cross multiplication. [Ex–11]
2. Numerical illustrations are not to be accepted in lieu of proofs; i.e., in proving $\sin A + \cot A \cdot \cos A = \csc A$, no credit is allowed if $m\angle A = 45$ is used.
3. Verbal problems having to do with the solution of triangles involve much more than mere computation. Credit should be divided between the analysis of the problem and the computation. The analysis should include the drawing of a correct figure, a statement of the formula or formulas to be used, and the formation of the equation or equations from which the unknown parts are to be obtained. A correctly labeled diagram should receive no more than 20 percent.
4. The common deductions for trigonometric graphs are:
   (a) Axes not labeled and/or x-axis not labeled in radians: –1 [Ex–18*a*]
   (b) Failure to label graphs when more than one is to be drawn on the same set of axes (no deduction if only one line or curve is not labeled): –1 [Ex–18*a*]
   (c) Graph drawn as a set of line segments: –1
5. Trigonometric graphs which are to be *sketched* should be done on appropriate graph paper; however, no deduction should be made for a correct, legible, and clearly labeled graph not done on graph paper.
6. For graphing a trigonometric equation worth a total of 4 credits, the deductions are:
   (a) Major flaw with the interval: –2 [Ex–18*a*]
   (b) Amplitude and frequency interchanged: –2 [Ex–18*a*]
   (c) Failure of graph to pass through such key points as maximum or minimum points and x- and y-intercepts: –1
7. The graph of $y = \sin x$ or $y = \cos x$ in place of a graph with an amplitude and/or frequency other than 1 should receive 1 credit.
8. A correct rational approximation, to at least three significant figures, for irrational values of trigonometric functions is an acceptable equivalent, unless otherwise specified.
9. Unless the measure of an angle is specifically requested in degrees or radians, either form may be accepted.

LOGIC

1. The credits assigned to a truth table are divided among the columns. Usually a simple negation column receives 1 credit, while a conjunction, implication, etc., should receive 2 credits. Each error in a column is a 1-credit deduction; however, the total deduction for any one column should never exceed the total credits assigned to that column. [Ex–16]
2. A logic proof should demonstrate a logically correct sequence of conclusions obtained by applying the laws of logic to premises and previously drawn conclusions. The form should be consistent with that presented by the instructor (two-column or narrative). In all cases, reasons must be provided for each

step of the proof. In no case should a mere designation of the truth values of the variables involved be construed as a proof. The symbolizing of the statements given should not receive more than 20 percent. [Ex–14(i),(ii),(iii),(iv)]

PROBABILITY AND STATISTICS

1. For Part I questions, where no partial credit is allowed, no credit should be allowed for answers left in ${}_nC_r$, $\binom{n}{r}$, ${}_nP_r$, or factorial notation. However, when partial credit is allowed, a deduction of 1 credit would apply for each answer left in this notation form. [Ex–12*c*]
2. Histograms should be drawn on graph paper using a straightedge. [Ex–19*b*]
3. The common deductions for graphs of frequency histograms or cumulative frequency histograms are as follows:
   (a) Failure to label the axes: –1
   (b) Failure to indicate uniform scale: –1
   (c) Drawing of a bar graph instead of a histogram: –1
   (d) Failure to use a straightedge: –1
   [Ex–19*b*]
4. Histograms are usually illustrated with an ascending horizontal scale; however, a descending horizontal scale is sometimes permissible. The graph drawn must be consistent with the table. Percentiles and quartiles must be consistent with the data given in the original question. [Ex–19*b*]
5. When rating standard deviation questions, allow no credit for the formula, since it appears on the formula sheet. In a set of grouped data, failure to use the frequencies is a major error and should result in a deduction of 50 percent. If work is shown, partial credit may be appropriate. Since some students may be using scientific calculators that have statistical features, the rater must decide what work needs to be shown for partial credit. [Ex–13(i),(ii),(iii)]

Sample Student Responses to Part I Questions

(No partial credit is allowed.)

1. Question

How many distinct six-letter permutations can be formed using the letters of the word "ALBANY"?

Key

360

Student Responses

(i) $\frac{6!}{2!}$ (+0)

(ii) $\frac{720}{2}$ (+2)

Rating Explanation

(i) Factorial form is not acceptable. [PROB-STAT-1]

(ii) Single fractions need not be in lowest terms unless the directions say to simplify or express in lowest terms. [ALG-1(a)]

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2. Question

If $f(x) = 4 \sin \frac{x}{3}$, find $f(\pi)$.

Key

$2\sqrt{3}$

Student Responses

(i) $4 \sin \frac{\pi}{3}$ (+0)

(ii) 3.4641 (+2)

(iii) 3.5 (+0)

Rating Explanation

(i) Order of operations cannot be determined. [ALG-1(f)]

(ii) Correct rational approximation to at least three significant figures [ALG-1(d); GEN-10]

(iii) Needs at least three significant figures [ALG-1(d); GEN-10]

3. Question

Find the negative root of the equation $|3x - 2| = 4$.

Key

$-\frac{2}{3}$

Student Response

$\{2, -\frac{2}{3}\}$ (+0)

Rating Explanation

Only the negative root was asked for and student does not differentiate between positive and negative root. [ALG–3(b)]

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4. Question

Solve for x:

$$x + y = 12$$
$$y = 3x$$

Key

3

Student Responses

(i) $(3,9)$ or $x = 3, y = 9$ (+2)

(ii) $(3,6)$ or $x = 3, y = 6$ (+0)

Rating Explanations

(i) Full credit when both answers are correct [ALG–3(a)]

(ii) Second value is not correct. [ALG–3(a)]

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5. Question

Find the slope of the line whose equation is $y = -\frac{2x}{3} + 2$.

Key

$-\frac{2}{3}$

Student Responses

(i) slope $= -\frac{2}{3}$
y-intercept $= 2$ (+2)

(ii) slope $= -\frac{2}{3}$
y-intercept $= 3$ (+0)

(iii) $m = -\frac{2}{3}$
$b = 2$ (+0)

(iv) $m = -\frac{2}{3}$ (+2)

Rating Explanations

(i) Both are correct and clearly identified. [ALG–3(c)]

(ii) y-intercept not correct [ALG–3(c)]

(iii) Slope not clearly identified [ALG–3(c)]

(iv) Acceptable answer

6. Question

Find the positive root of $x^2 - 49 = 0$.

Key

7

Student Response

± 7 (+0)

Rating Explanation

Only the positive root was asked for. [ALG-3(b)]

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7. Question

Write an equation of the line whose slope is $\frac{2}{3}$ and which passes through the point (6,−1).

Key

$y = \frac{2}{3}x - 5$

Student Responses

(i) $y + 1 = \frac{2}{3}(x - 6)$ (+2)

(ii) $2x - 3y = 15$ (+2)

Rating Explanations

(i) An equivalent equation [GRAPH-10]

(ii) Same as (i) [GRAPH-10]

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8. Question

If $a * b$ is a binary operation defined as $a + 2b$, evaluate $5 * 3$.

Key

11

Student Response

$5 + 2(3)$ (+0)

Rating Explanation

Result depends upon the proper use of the order of operations. [ALG-1(f)]

9. Question

Express, in radians, the angle in Quadrant II whose sine is $\frac{\sqrt{3}}{2}$.

Key

$\frac{2\pi}{3}$

Student Response

2.09

(+2)

Rating Explanation

Correct rational approximation of $\frac{2\pi}{3}$ to at least three significant figures [GEOM–9]

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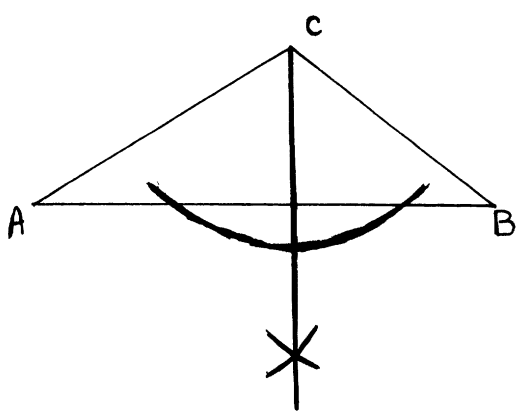
10. Question

On the answer sheet, construct the altitude to side $\overline{AB}$ of $\triangle ABC$.

Student Response



(+2)

Rating Explanation

The altitude required is a subset of the line necessary for the construction. [GEOM–1]

Sample Student Responses to Part II or Part III Questions

(Partial credit is allowed.)

Be sure to note the credit of each example, since many are not full 10-point Part II/III questions.

11. Question

For all values of θ for which the expressions are defined, show that the following is an identity:

$$\frac{\sin 2\theta}{\tan\theta} = \frac{2}{1 + \tan^2\theta} \quad [5]$$

Student Response

$$\frac{\sin 2\theta}{\tan\theta} = \frac{2}{1+\tan^2\theta} \qquad (+1)$$

$$\sin 2\theta(1+\tan^2\theta) = 2\tan\theta$$

$$2\sin\theta\cos\theta(\sec^2\theta) = \frac{2\sin\theta}{\cos\theta}$$

$$\frac{2\sin\theta}{\cos\theta} = \frac{2\sin\theta}{\cos\theta}$$

Rating Explanation

Cross multiplication results in a 50 percent deduction. (–3)
One credit is deducted for work not shown. In Course III, formulas are given; therefore, credit is only allowed for substitution. [CIR FN–1]

12. Question

A florist has 5 red roses, 3 pink roses, and 2 yellow roses.

a How many different bouquets can be made using 3 roses? [2]

b How many of these bouquets can be formed using 3 red roses? [2]

c How many of these bouquets can be formed using 2 pink roses and 1 yellow rose? [2]

d What is the probability that one of these bouquets consists of 3 red roses? [2]

e What is the probability that one of these bouquets consists of 2 pink roses and 1 yellow rose? [2]

Key

a 120

b 10

c 6

d $\frac{1}{12}$

e $\frac{1}{20}$

Student Response		Rating Explanation
a $_{10}C_3 = \frac{10!}{3!7!} = 120$	(+2)	*a* correct
b $_5C_3 = \frac{5!}{3!2!} = 10$	(+2)	*b* correct
c $_3C_2 \cdot {_2C_1}$	(+1)	*c* did not simplify [PROB–STAT–1]
d $\frac{10}{120} = \frac{1}{12} = 12\%$	(+1)	*d* additional incorrect information given [GEN–23]
e $\frac{_3C_2 \cdot {_2C_1}}{120}$	(+2)	*e* correct, based on answers in *a* and *c* [GEN–12]

13. Question

Find the mean and find, to the *nearest tenth*, the standard deviation for the following data. [4]

Score (x_i)	Frequency (f_i)
20	3
22	2
23	2
25	2
30	1

Key

mean = 23
standard deviation = 2.9

Student Responses

(i) $\bar{x} = 23$, s.d. = 2.9 (+4)

(ii) $\bar{x} = 24$, s.d. = 3.4 (+0)

(iii) (+2)

x_i	f_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
20	3	−4	16
22	2	−2	4
23	2	−1	1
25	2	1	1
30	1	6	36

$\frac{120}{5} = 24$ $\frac{58}{5} = 11.6$

S.D. = $\sqrt{11.6}$ = 3.406 = 3.4

Rating Explanations

(i) Work may not be necessary as some calculators have statistical functions. [PROB–STAT–5]

(ii) No work shown to support answer [PROB–STAT–5]

(iii) Since the work is shown, 50 percent (2 credits) is deducted for ignoring frequencies. [PROB–STAT–5]

14. Question

Given:

Janice is smart.
If Janice is smart and has a job, then she will earn a good income.
If Janice works hard, then she will have a job.
If Janice does not work hard, then she will be unhappy.
Janice does not have a good income.

Let *S* represent:
"Janice is smart."
Let *W* represent:
"Janice works hard."
Let *J* represent:
"She has a job."
Let *I* represent:
"She will earn a good income."
Let *U* represent:
"She will be unhappy."

Prove: Janice will be unhappy. [10]

Student Responses

(i)

$\overset{T}{S}$

$(\overset{T}{S} \wedge \overset{F}{J}) \rightarrow \overset{F}{I}$

$\overset{F}{W} \rightarrow \overset{F}{J}$

$\sim\overset{T}{W} \rightarrow \overset{T}{U}$

$\sim\overset{T}{I}$

U is true.

(+2)

Rating Explanations

(i) Mere designation of the truth values is not a proof. (–8)

Symbolizing only receives a maximum of 20 percent of the credit. [LOGIC–2]

(ii)

S

$(S \wedge J) \to I$

$W \to J$

$\sim W \to U$

$\sim I$

$([(S \wedge J) \to I] \wedge \sim I) \to \sim(S \wedge J)$

$\sim(S \wedge J) \leftrightarrow \sim S \vee \sim J$

$[(\sim S \vee \sim J) \wedge S] \to \sim J$

$[(W \to J) \wedge \sim J] \to \sim W$

$[(\sim W \to U) \wedge \sim W] \to U$

QED

(+2)

(+4)

(TOTAL: +6)

(ii) Symbolizing of premises

Allow only 50 percent of credit because no reasons are given. [LOGIC–2]

(iii)

S

$(S \wedge J) \to I$

$W \to J$

$\sim W \to U$

$\sim I$

$(S \wedge J) \to I$

$\sim I$

$\therefore \sim S \vee \sim J$

S

$\therefore \sim J$

$W \to J$

$\therefore \sim W$

$\sim W \to U$

$\therefore U$

(+2)

(TOTAL: +5)

(iii) Symbolizing

No reasons are given. (–4)

Transition from $\sim(S \wedge J)$ to $\sim S \vee \sim J$ is not shown. (–1) [LOGIC–2; GEN–21]

(iv)

Statements	Reasons
1. S	1. Given
2. $(S \wedge J) \rightarrow I$	2. "
3. $W \rightarrow J$	3. "
4. $\sim W \rightarrow U$	4. "
5. $\sim I$	5. "
6. $\sim(S \wedge J)$	6. law of contrapositive inference (2,5)
7. $\sim S \vee \sim J$	7. De Morgan's laws (6)
8. J	8. same as 6 (1, 7)
9. W	9. law of converse (3, 8)
10. U	10. law of inverse (4, 9)

(iv) steps 1–5
Correct symbolizing of arguments (+2)

step 6
If the form is consistent with the method used by the instructor, no deduction should be made. (+2)

step 7
same as 6 (+1)

steps 8–10
All are invalid conclusions. (+0) [LOGIC–2; GEOM–5]

(TOTAL: +5)

15. Question

Using the accompanying table, solve for all x:

$$5 * x = 5 \quad [2]$$

*	1	3	5	7
1	1	1	1	1
3	1	3	3	3
5	1	3	5	5
7	1	3	5	7

Key

{5,7}

Student Responses		Rating Explanations
(i) {5}	(+1)	(i) 1 credit deducted for omitted answer
(ii) {7}	(+1)	(ii) same as (i)
(iii) {1,5}	(+1)	(iii) 1 credit deducted for an incorrect answer
(iv) {1,5,7}	(+1)	(iv) same as (iii)
(v) {1,3,5,7}	(+0)	(v) 2 credits deducted for incorrect answers
(vi) {1,3,5}	(+0)	(vi) same as (v) [ALG–13]

16. Question

On your answer paper, construct a truth table for the statement
$\sim(p \rightarrow \sim q) \longleftrightarrow (p \wedge q)$. [10]

Student Response

p	q	$\sim q$	$p \wedge q$	$p \rightarrow \sim q$	$\sim(p \rightarrow \sim q) \leftrightarrow (p \wedge q)$
T	T	F	T	F	T
T	F	T	T	T	F
F	T	F	T	T	F
F	F	T	F	T	F
(+1)	(+1)	(+1)	(+0)	(+2)	(+1)

(TOTAL: +6)

Rating Explanation

Fourth column has two mistakes. (-2) [LOGIC-1]
Either the last column heading is correct and the first three entries are correct based on incorrect earlier responses or the student has left out the $\sim(p \rightarrow \sim q) \longleftrightarrow (p \wedge q)$ column altogether. [LOGIC-1; GEN-12]

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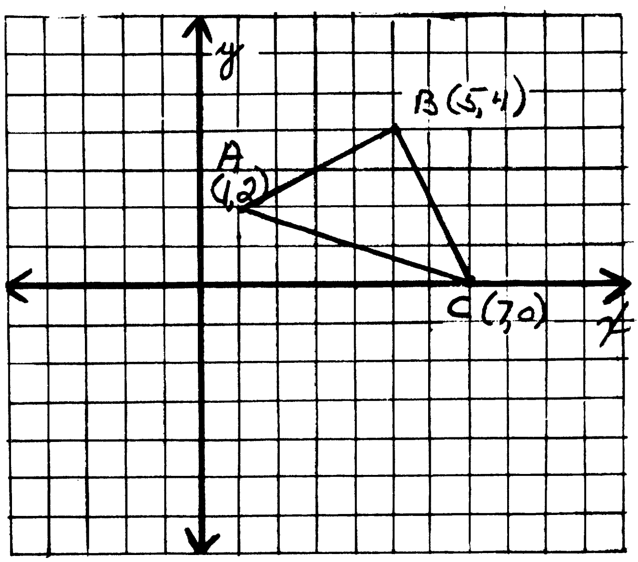
17. Question

The vertices of ΔABC are $A(1,2)$, $B(5,4)$, and $C(7,0)$. Prove that ΔABC is an isosceles right triangle. [10]

Student Response



Rating Explanation

Conclusions with appropriate reasons are not given. [GRAPH-8]

$AB = \sqrt{16+4} = \sqrt{20}$

$BC = \sqrt{4+16} = \sqrt{20}$

$AC = \sqrt{36+4} = \sqrt{40}$ (+3)

$(\sqrt{20})^2 + (\sqrt{20})^2 = (\sqrt{40})^2$ (+3)

$20 + 20 = 40$ (TOTAL: +6)

18. Question

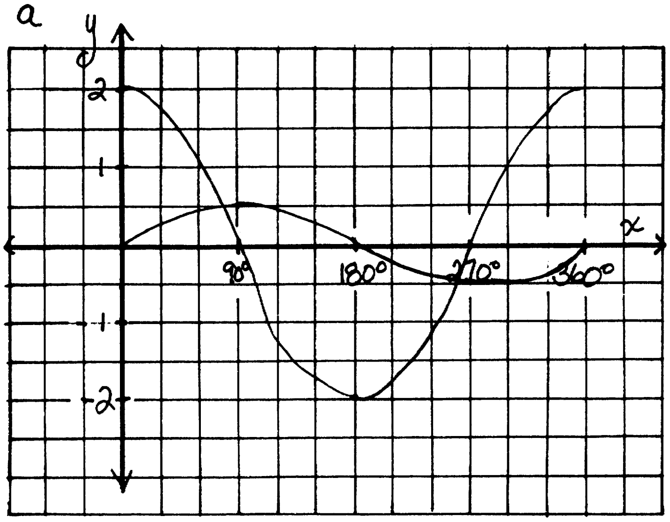
a On the same set of axes, sketch and label the graphs of $y = \sin \frac{1}{2}x$ and $y = 2 \cos x$ as x varies from $-\pi$ to π radians. [8]

b From the graphs made in part *a*, how many values of x in the interval $-\pi \leq x \leq \pi$ satisfy the equation $\sin \frac{1}{2}x = 2 \cos x$? [2]

Key

Student Response

a



b 2

(TOTAL: +4)

Rating Explanations

The graph of $y = \sin \frac{1}{2}x$ has the amplitude and frequency transposed. (–2) [CIR FN–6(b)]
The x-axis is labeled in degrees, not radians, and axes are not labeled. (–1) [CIR FN–4(a)]
Curves are not labeled. (–1) [CIR FN–4(b)]
Curves are not drawn over the proper interval. (–2) [CIR FN–6(a)]
No credit is deducted on part *b*, since answer is correct based on the curves drawn in part *a*. [GEN–12]

19. Question

For a class project, 20 students recorded the number of hours of television that they each watched in one week: 5, 12, 29, 23, 35, 8, 41, 40, 13, 16, 31, 29, 18, 28, 15, 32, 38, 26, 20, 22.

a *On your answer sheet*, copy and complete the tables below to find the frequency and cumulative frequency in each interval. [4]

Interval	Tally	Frequency
0–9		
10–19		
20–29		
30–39		
40–49		

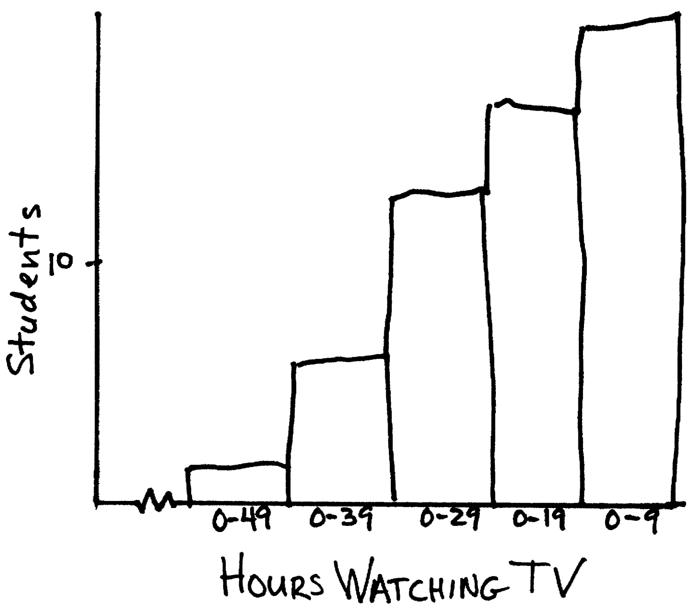
Interval	Cumulative Frequency
0–9	
0–19	
0–29	
0–39	
0–49	

b Using the cumulative frequency table completed in part *a*, construct a cumulative frequency histogram. [4]

Student Response

Interval	Tally	Frequency
0-9	II	2
10-19	~~IIII~~	5
20-29	~~IIII~~ II	7
30-39	IIII	4
40-49	II	2

Interval	Cumulative Frequency
0-9	20
0-19	18
0-29	13
0-39	6
0-49	2



(TOTAL: +5)

Rating Explanation

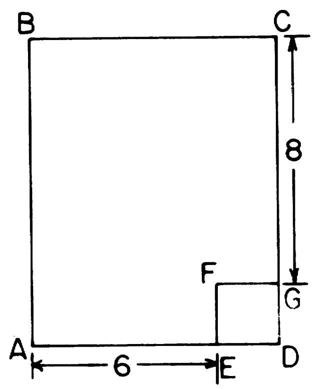
(+2) *a* Tally and frequencies correct

Major error with cumulative frequency (-2)

(+3) *b* Graph is correct based upon the cumulative frequency chart in part *a*. [PROB-STAT-4; GEN-12] Freehand drawing is inaccurate. (-1) [PROB-STAT-2,3(d)]

20. Question

In the accompanying diagram, *ABCD* is a rectangle and *DEFG* is a square. The area of *ABCD* is 80, *CG* = 8, and *AE* = 6. Find the length of the side of square *DEFG*. [*Only an algebraic solution will be accepted.*] [5,5]



Key

2

Student Response

Let x = the length of the side of the square

$7 \cdot 9 = 63$

$8 \cdot 10 = 80$ ✓

$6 + x = 8$ $\quad$ $6 + 2 = 8$

$8 + x = 10$ $\quad$ $8 + 2 = 10$

So $x = 2$ and the side of the square is 2.

(+3)

Rating Explanation

One point is given for the analysis (representing the unknown) and 2 points for the answer. No other credit is allowed because question specified an algebraic solution. [ALG–5,6,9; GEN–15]

21. Question

Find the area of pentagon $ABCDE$, whose vertices are $A(-2,-5)$, $B(-2,2)$, $C(2,4)$, $D(5,2)$, and $E(4,-2)$. [10]

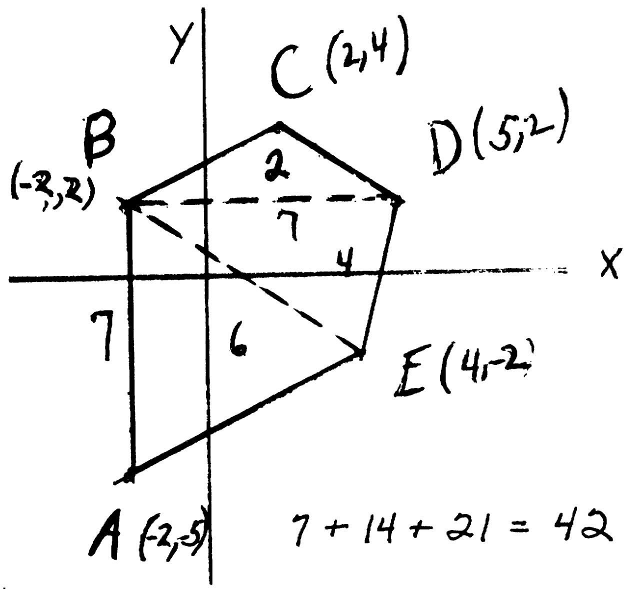
Key

42

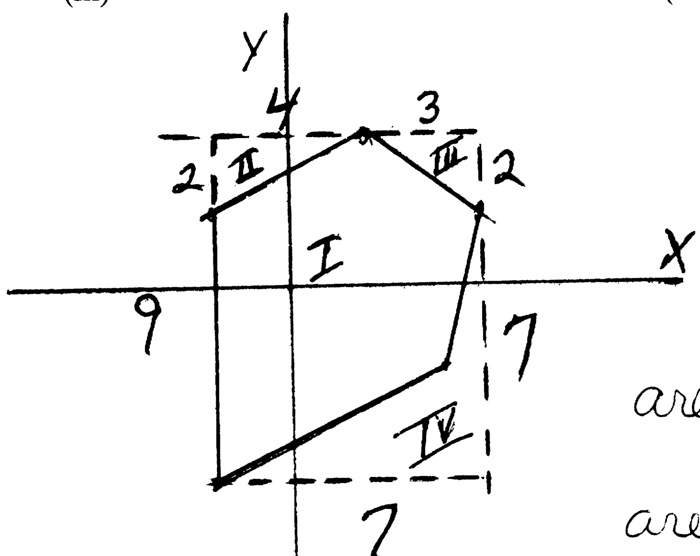
Student Responses		**Rating Explanations**
(i) 42	(+2)	(i) answer only [GRAPH–7(a)]
(ii) (see graph below)	(+10)	(ii) Since the method of solution can be clearly determined from the graph and since mental arithmetic and a simple formula can be used, full credit is granted. [GRAPH–7(b); GEN-13,21]
(iii) (see graph below)	(+6)	(iii) Improper analysis, since region IV is not a triangle (-4) [GEN–14]

(i) 42

(ii)



(iii)



area I = area of rectangle – (area II + area III + area IV)

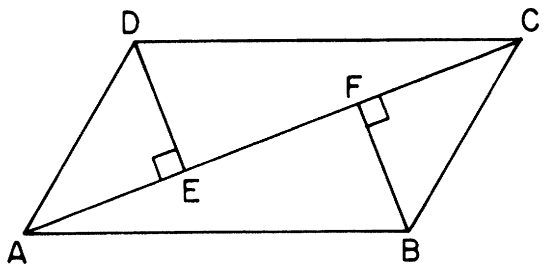
area I = 63 – (4 + 3 + 24½)

= 63 – 31½

= 31½

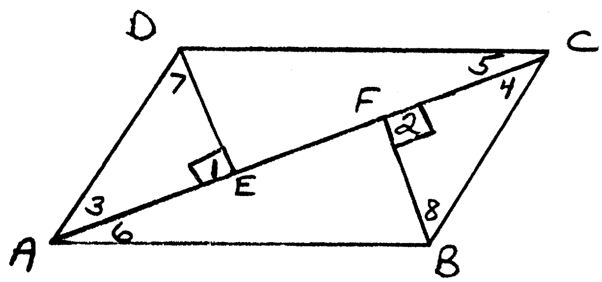
22. Question

Given: quadrilateral *ABCD*, diagonal $\overline{AEFC}$, $\overline{DE} \perp \overline{AC}$, $\overline{BF} \perp \overline{AC}$, $\overline{AE} \cong \overline{CF}$, and $\overline{DE} \cong \overline{BF}$.



Prove: *ABCD* is a parallelogram. [10]

Student Response



Prove: ABCD is a ▱

Statements	Reasons
1. diagonal $\overline{AEFC}$, $\overline{DE} \perp \overline{AC}$, $BF \perp \overline{AC}$, $\overline{AE} \cong \overline{CF}$, $\overline{DE} \cong \overline{BF}$	1. Given
2. $\angle 1 \cong \angle 2$	2. All rt. angles are $\cong$
3. $\triangle ADE \cong CBF$	3, SAS
4. $\angle 3 \cong \angle 4$, $\angle 7 \cong \angle 8$, $\overline{AD} \cong \overline{CB}$	4. CPCTC
5. $\overline{AC} \cong \overline{CA}$	5, Identity
6. $\triangle DAC \cong \triangle BCA$	6. SAS
7. $\angle 5 \cong \angle 6$, $\overline{DC} \cong \overline{BA}$	7, CPCTC
8. $\overline{DC} \parallel \overline{AB}$, $\overline{AD} \parallel \overline{BC}$	8. $\cong$ corresponding angles make $\parallel$ lines
9. ABCD is a ▱	9. If both pairs of opposite sides of a quad. are $\cong$, the quad. is a ▱.

Rating Explanation

(+8) Credit given assumes that the form of the reasons is consistent with that presented by the instructor. [GEOM–4]

$\angle 1$ and $\angle 2$ are not identified as right angles in the given information. (-1) [GEOM–5; GEN–21]

In step 4, $\angle 7 \cong \angle 8$ is irrelevant correct information which does not detract from the nature of the proof. [GEN–23]

Step 8 contains irrelevant incorrect information (the reason). (-1) [GEN–23]

23. Question

Given the equation: $y = -x^2 + 4x - 3$
Draw the graph of the equation using all values of x such that $-2 \leq x \leq 6$. [6]

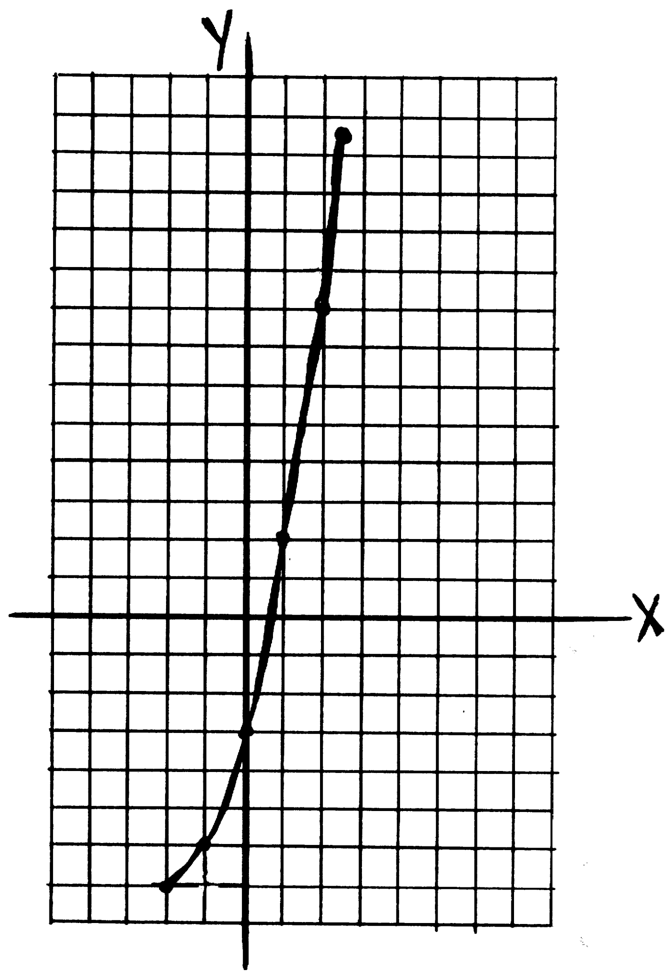
Student Response

$y = -x^2 + 4x - 3$

x	y
-2	$4 - 8 - 3 = -7$
-1	$1 - 4 - 3 = -6$
0	$-3 = -3$
1	$1 + 4 - 3 = 2$
2	$4 + 8 - 3 = 9$
3	$9 + 12 - 3 = 18$
4	$16 + 16 - 3 = 29$

Rating Explanation

(+2) Improper evaluation of $-x^2$ is a major error in principle. (−3) [GEN–14]
Failure to graph or even complete the table over the entire interval (−1) [GRAPH–2(d)]



24. Question

Mary chose one of the four numbers 1, 2, 3, and 6 at random. She then chose one of the two numbers 1 and 5 at random.

a Draw a tree diagram or list the sample space of all possible pairs of numbers that Mary could choose. [3]

b Find the probability that Mary chose an even number first followed by an odd number. [2]

c Find the probability that Mary chose *at least* one even number. [2]

d Find the probability that both choices were the same number. [2]

e Find the probability that Mary chose two even numbers. [1]

Key

b $\frac{4}{8}$

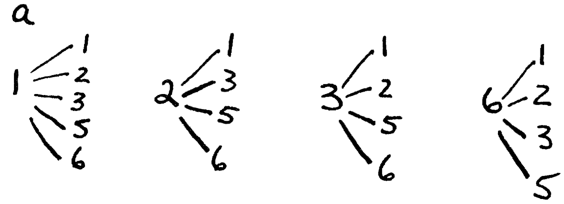
c $\frac{4}{8}$

d $\frac{1}{8}$

e 0

Student Response

a



(+1)

b $\frac{6}{17}$

c $\frac{12}{17}$

(+4)

d $\frac{2}{17}$

(+0)

e 0

Rating Explanation

a An understanding of the use of tree diagrams for two choices is shown; however, second choice options are incorrect. (-2) [GEN–14]

b,c Correct for the information in part *a* [GEN–12]

d,e Incorrect for diagram used in part *a* (-4) [GEN–12]

25. Question

In ΔABC, $m\angle C = 90$, $AC = x$, $BC = x + 3$, and $AB = x + 5$.

a Write an equation in terms of x which can be used to find AC. [3]

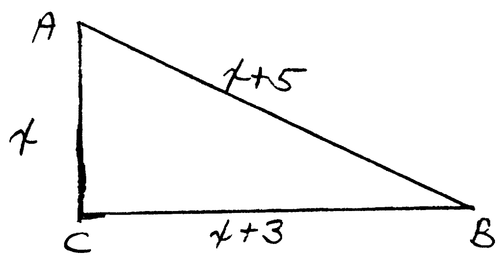
b Find AC. [*Answer may be left in radical form.*] [7]

Key

a $x^2 + (x + 3)^2 = (x + 5)^2$

b $2 + 2\sqrt{5}$ *or* $\dfrac{4 + \sqrt{80}}{4}$

Student Response



a $2x^2+6x+9=x^2+10x+25$

b $x^2-4x+16=0$

$$x = \frac{4 \pm \sqrt{16-4(1)(16)}}{2}$$

Rating Explanation

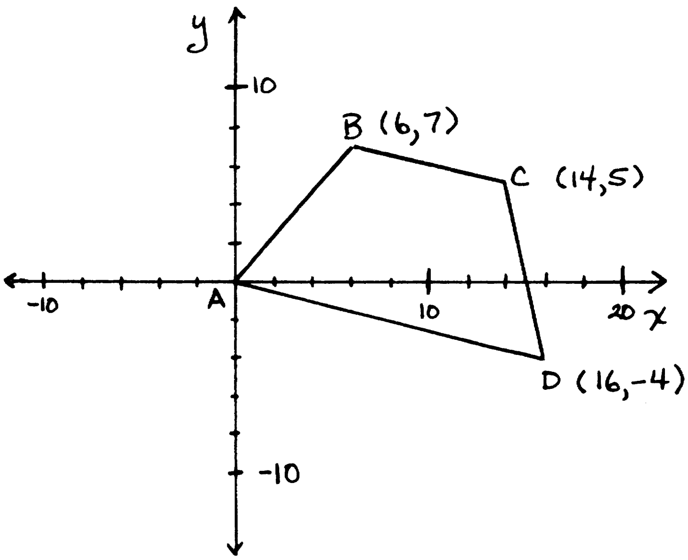
(+3) *a* Part *a* has an answer which is algebraically equivalent to the given answer. [GEN–10]

(+4) *b* Correct squaring and combining of similar terms [GEOM–8]
Computational error (+16 instead of −16) (−1) [GEN–14]
Correct substitution into formula [GEOM–8]
Simplification and elimination of extraneous roots omitted. (−2) [ALG–10]

26. Question

The vertices of a quadrilateral are $A(0,0)$, $B(6,7)$, $C(14,5)$, and $D(16,-4)$. Prove that $ABCD$ is an isosceles trapezoid. [10]

Student Response



$m_{\overline{BC}} = \frac{5-7}{14-6} = \frac{-2}{8} = -\frac{1}{4}$

$m_{\overline{AD}} = \frac{-4}{16} = -\frac{1}{4}$

$\therefore$ $\overline{AD} \parallel \overline{BC}$ since lines with equal slopes are $\parallel$

$\therefore$ ABCD is a trapezoid because it has a pair of $\parallel$ sides

$AB = \sqrt{36+49} = \sqrt{85}$

$CD = \sqrt{(16-14)^2+(-4-5)^2} = \sqrt{2+81} = \sqrt{83}$

AB and CD aren't equal so ABCD is not isosceles.

Rating Explanation

(+6) Slopes of $\overline{BC}$ and $\overline{AD}$ are correct and reason is given for the lines being parallel. [GRAPH–8]
It has not been shown that $\overline{AB}$ is not parallel to $\overline{CD}$. (-3) [GEOM–4; GEN–14]
Computation error in calculating the length of $\overline{CD}$ (-1) [GEN–14]
The correct calculation of the length of $\overline{AB}$, together with an appropriate conclusion drawn, based on these calculations [GEN–12]

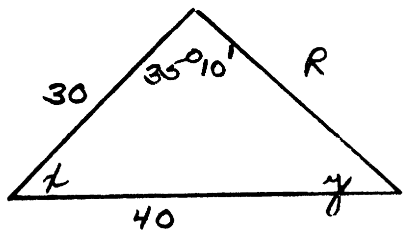
27. Question

Two forces of 30 pounds and 40 pounds act upon a body, forming an acute angle with each other. The angle between the resultant and the 30-pound force is 35°10′. Find, to the *nearest ten minutes or nearest hundredth of a degree*, the angle between the two given forces. [6]

Key

60°50′ *or* 60.76°

Student Response



$$\frac{40}{\sin 35°10'} = \frac{30}{\sin y}$$

$y = 25.592495°$

$y = 25° 40'$

$x = 119° 10'$

Rating Explanation

Major error in interpretation (–3)

(+3) Correct value of y based on improper analysis and correct value of x based on y [GEN–13]

28. Question

Find, to the *nearest degree*, all values of θ in the interval $0° \leq \theta \leq 360°$ which satisfy the equation $3 \cos 2\theta + \sin \theta - 2 = 0$. [10]

Key

30°, 150°, 199°, 341°

Student Response

(+7)

$$3 \cos 2\theta + \sin\theta - 2 = 0$$
$$3(1 - 2\sin^2\theta) + \sin\theta - 2 = 0$$
$$-6\sin^2\theta + \sin\theta + 1 = 0$$
$$6\sin^2\theta - \sin\theta - 1 = 0$$
$$(3\sin\theta - 1)(2\sin\theta + 1) = 0$$

$\sin\theta = \frac{1}{3}$ $\quad$ $\sin\theta = -\frac{1}{2}$

$20°, 160°$ $\quad$ $\theta = -30°$

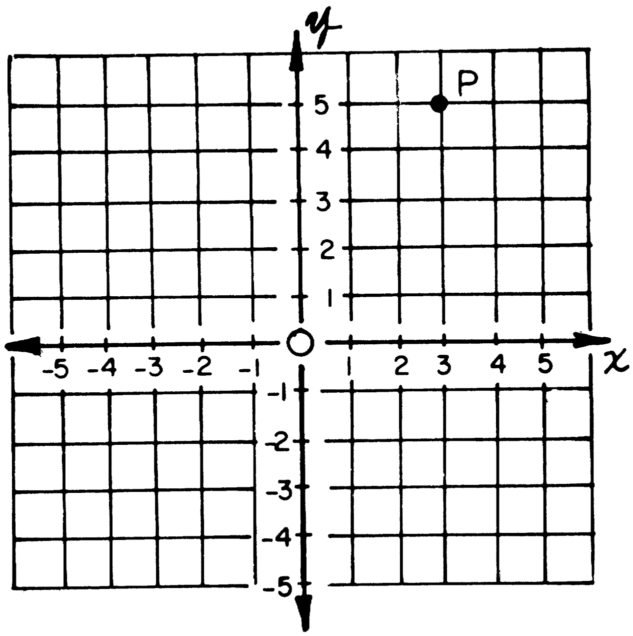
$210°, 330°$

Rating Explanation

Substitution and setting equal to zero
Factored incorrectly (–2) [GEN–14]
Correct values of sin θ based on factors used [GEN–12]
Incorrect rounding of angle for one pair of answers (–1) [GEN–14]
Correct angle measure for other pair of answers
Note that the scientific calculator generated an angle of –30° for Arc sin $(-\frac{1}{2})$, but the domain requires the answers of 210° and 330°. [GEN–12]
Credit for solution is granted because the difficulty level is equivalent to that of the correct solution. [GEN–18]

29. Question

In the accompanying graph, P is a point whose coordinates are (3,5).



a Describe fully the locus of points a distance of d units from P. [3]

b Describe fully the locus of points a distance of one unit from the y-axis. [4]

c How many points satisfy the conditions in parts *a* and *b* simultaneously for the following values of d?
(1) $d = 2$ [1]
(2) $d = 4$ [1]
(3) $d = 5$ [1]

Key

a A circle with center P and radius d

b Two vertical lines, each one unit from the y-axis

c (1) 1
(2) 3
(3) 4

Student Response

a a circle (+1)

b the line x = 1 (+2)

c (1) 1 (+3)
(2) 2
(3) 2

Rating Explanation

a Locus is not fully described. (–2) [GEN–16]

b Same as *a* (–2) [GEN–16]

c The rest is correct based on the answers in parts *a* and *b*. [GEN–12]

30. Question

Solve for x to the *nearest tenth*:

$15^x = 0.15$ [4]

Key

-0.7

Student Responses

(i) $-.7$ (+4)

(ii) $x \log 15 = \log .15$

$x = -.7$ (+4)

(iii) $x \log 15 = \log .15$ (+2)

$= \log .15 - \log 15$

$= -.8239 - 1.1761$

$= .3521$

$= .4$

Rating Explanations

(i) Correct solution, if teacher permits [GEN-13]

(ii) Correct solution [GEN-27]

(iii) Correct logarithm theory; note the negative logarithm for 0.15.

Additional incorrect theory (-1) [GEN-23]

Calculation error (-1) [GEN-14]

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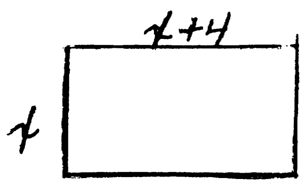
31. Question

The length of a rectangle is 4 more than its width. If the area is 32, find the dimensions of the rectangle. [5,5]

Key

4,8

Student Response



$A = 2w + 2l$

$32 = 2(x) + 2(x+4)$

$32 = 2x + 2x + 8$

$32 = 4x + 8$

$24 = 4x$

$6 = x$

$10 = x + 4$

Rating Explanation

(+2) Variables are represented in the diagram. [ALG-6]
Use of the wrong formula is a major error. (-3) [GEN-14]
Solving a much easier equation (-4) [GEN-18]

(+1) Correct second dimension is found.

32. Question

Solve and check:

$$y = 2x$$
$$y = 2x^2 + 3x - 1 \quad [8,2]$$

Key

$(\frac{1}{2},1)$ and $(-1,-2)$

or

$x = \frac{1}{2}$, $y = 1$ and $x = -1$, $y = -2$

Student Response

$2x = 2x^2 + 3x - 1$

$2x^2 + x - 1 = 0$

$(2x + 1)(x - 1) = 0$

$2x + 1 = 0 \qquad x = 1$

$x = -\frac{1}{2} \qquad y = 2(1) = 2$

$y = 2(-\frac{1}{2}) = -1$

$-1 = 2(-\frac{1}{2}) \qquad 2 = 2(1)$

Rating Explanation

(+6) Correct substitution
Equation is correctly set equal to zero.
Incorrect factoring (-2)
Solved the incorrect equations correctly for x and y [GEN-12]

(+0) Check only involves substituting into the equation from which the second unknown was determined. (-2) [ALG-12]

33. Question

a On the same set of coordinate axes, graph the following system of inequalities:

$$\begin{array}{l} y < -3x + 5 \\ x - y \geq 2 \end{array} \quad [8]$$

b Write the coordinates of a point in the solution set of the graph drawn in answer to part *a*. [2]

Student Response

a

$y < -3x + 5$

$m = -3$

$b = 5$

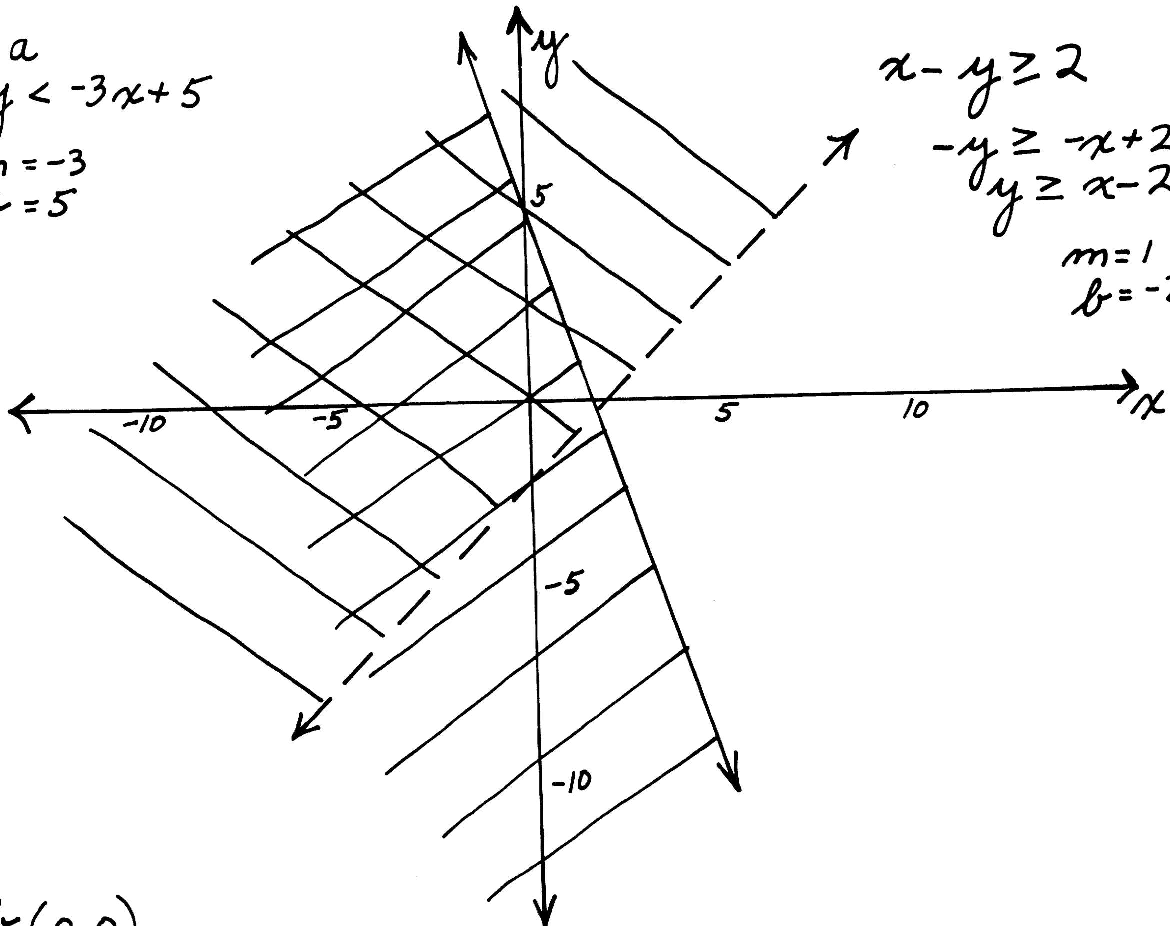
$x - y \geq 2$

$-y \geq -x + 2$

$y \geq x - 2$

$m = 1$

$b = -2$



b (0,0)

Rating Explanation

(+4) *a* The student has (due to an algebraic error) shaded the wrong side of the line for the second inequality. (−1) [GRAPH−4(b)]
The lines are not labeled. (−1) [GRAPH−2(b)]
The dotted line should be solid. (−1) [GRAPH−2(c)]
The solid line should be dotted. (−1) [GRAPH−2(c)]

(+2) *b* Part *b* is done correctly for the graph shown. [GEN−12]

34. Question

Find the three largest consecutive integers whose sum is less than 86. [*Only an algebraic solution will be accepted.*] [5,5]

Key

27, 28, 29

Student Responses

(i)

Let $x = 1^{st}$
$x+1 = 2^{nd}$
$x+2 = 3^{rd}$

$x+x+1+x+2 = 86$
$3x+3 = 86$
$3x = 83$
$x = 27\frac{2}{3}$

#'s are 27, 28, 29

(ii)

Let $x = 1^{st}$
$x+1 = 2^{nd}$
$x+2 = 3^{rd}$

$x+x+1+x+2 = 86$
$3x+3 = 86$
$3x = 83$
$x = 27\frac{2}{3}$
$x+1 = 28\frac{2}{3}$
$x+2 = 29\frac{2}{3}$

Rating Explanations

(+10) (i) Writing of an equation instead of an inequality is not an error, since the student has demonstrated a correct interpretation of the problem. [ALG–5,8]

(+6) (ii) Answers given do not demonstrate a correct interpretation of the problem. (–4) [ALG–5,8]

35. Question

The coordinates of the vertices of $\triangle ABC$ are $A(1,6)$, $B(2,9)$, and $C(7,10)$.

a Graph and label $\triangle ABC$. [1]

b Graph and state the coordinates of $\triangle A'B'C'$, the image of $\triangle ABC$ after a reflection over the line $y = x$. [3]

c Graph and state the coordinates of $\triangle A''B''C''$, the image of $\triangle A'B'C'$ after a reflection over the x-axis. [3]

d Graph and state the coordinates of $\triangle A'''B'''C'''$, the image of $\triangle A''B''C''$ after the transformation $(x,y) \rightarrow (x-5,y+3)$. [3]

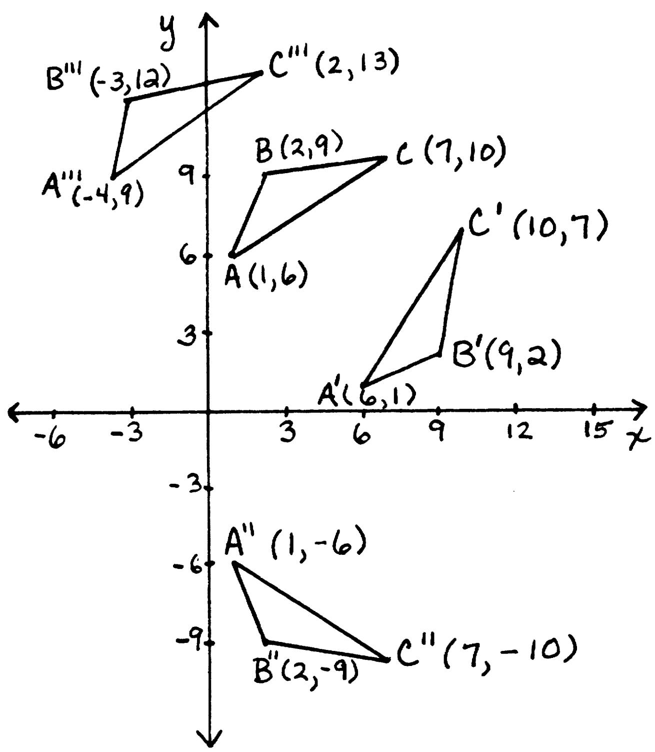
Key

b $A'(6,1)$, $B'(9,2)$, $C'(10,7)$

c $A''(6,-1)$, $B''(9,-2)$, $C''(10,-7)$

d $A'''(1,2)$, $B'''(4,1)$, $C'''(5,-4)$

Student Response



Rating Explanation

(+1) *a* Correct

(+3) *b* Correct

(+2) *c* Use of $\triangle ABC$ as pre-image, instead of $\triangle A'B'C'$, is not a cardinal error in principle. (–1) [GRAPH–11]

(+3) *d* $\triangle ABC$ was again used as the pre-image; however a deduction was already made in part *c*, and the student should not be penalized twice for the same mistake. [GRAPH–11]

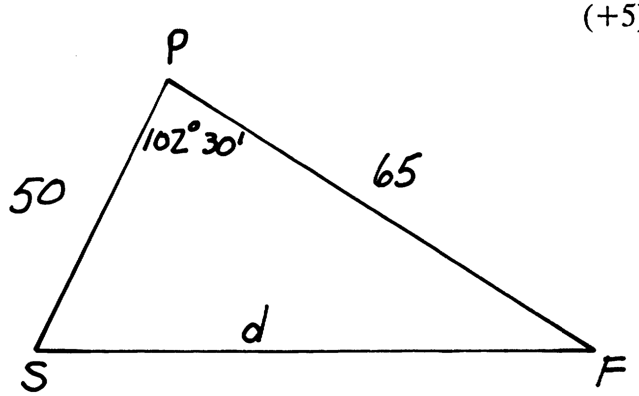
36. Question

Sailboat S is 50 meters from a lighthouse located at point P. Fishing boat F is 65 meters from the same lighthouse. If the measure of $\angle SPF$ is $102°30'$, find to the *nearest meter*, the distance between the two boats. [6]

Key

90

Student Response



$d^2 = 50^2 + 65^2 - 2(50)(65)\cos 102°30'$

$d = 90.2$

Rating Explanation

(+5) Correct analysis and substitution [GEN–27]
Incorrect level of accuracy (–1) [GEN–19]

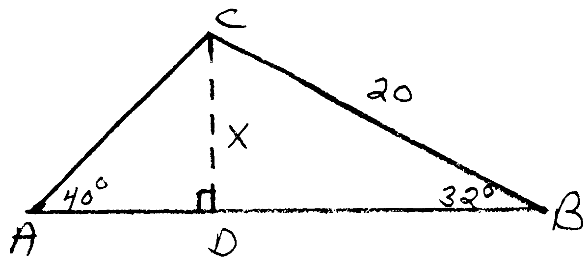
37. Question

In $\triangle ABC$, m$\angle A = 40$, m$\angle B = 32$, and $BC = 20$ meters. Find the length of AC to the *nearest meter*. [6]

Key

16

Student Response



$$\frac{X}{20} = \sin 32^\circ$$

$$X = 20 \sin 32^\circ$$

$$= 10.6$$

$$\frac{10.6}{AC} = \sin 40^\circ$$

$$AC = \frac{10.6}{\sin 40^\circ}$$

$$AC = 16.49$$

ans: 16

Rating Explanation

(+6) This problem could be solved more directly by using the law of sines; however, the work is correct and mathematically sound, so full credit is granted. [GEN–22,27]

38. Question

A wire that runs from the top of a vertical pole to the ground makes an angle of 41° with the ground. The wire touches the ground at a point 47.5 feet from the base of the pole.

a Find the height of the pole to the *nearest tenth* of a foot. [5]

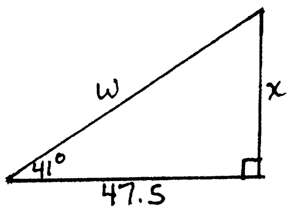
b Find the length of the wire to the *nearest tenth* of a foot. [5]

Key

a 41.3

b 62.9

Student Response



a $\tan 41° = \frac{x}{47.5}$

$x = 47.5 \tan 41°$

$x = 41.3$

b $\cos 41° = \frac{47.5}{w}$

$w = 35.8$

Rating Explanation

(+5) *a* Correct solution [GEN–27]

(+4) *b* Correct analysis
Calculations were incorrect. (–1)
[GEN–13]

39. Question

Solve the following system of equations graphically and check:

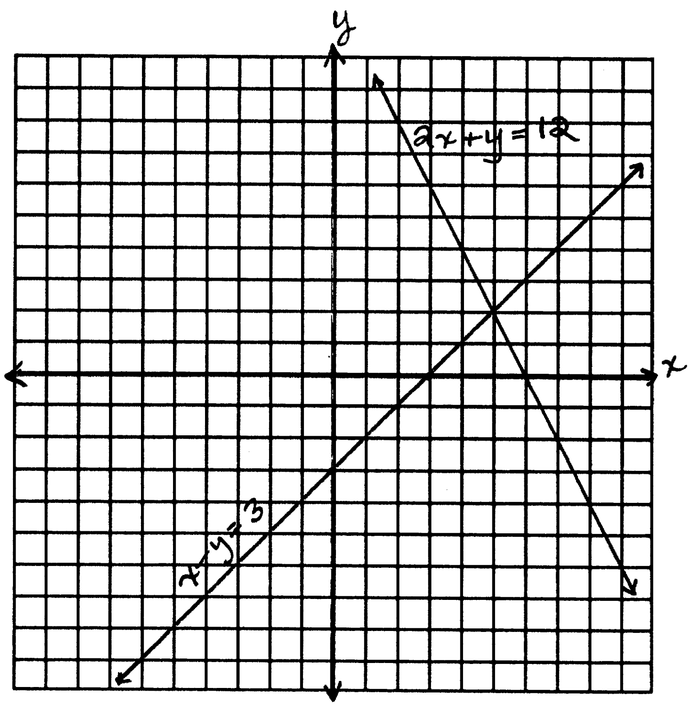
$$2x + y = 12$$
$$x - y = 3 \quad [8,2]$$

Key

$x = 5$
$y = 2$ *or* (5,2)

Student Responses

(i)



Check (5,2)
$2(5) + 2 = 12$
$10 + 2 = 12$ ✓
$5 - 2 = 3$ ✓

Rating Explanations

(+6) Correct lines
(+2) Solution is indicated by the correct check. [GRAPH–3(b)]
(+2) Correct checks

(ii) (+6)

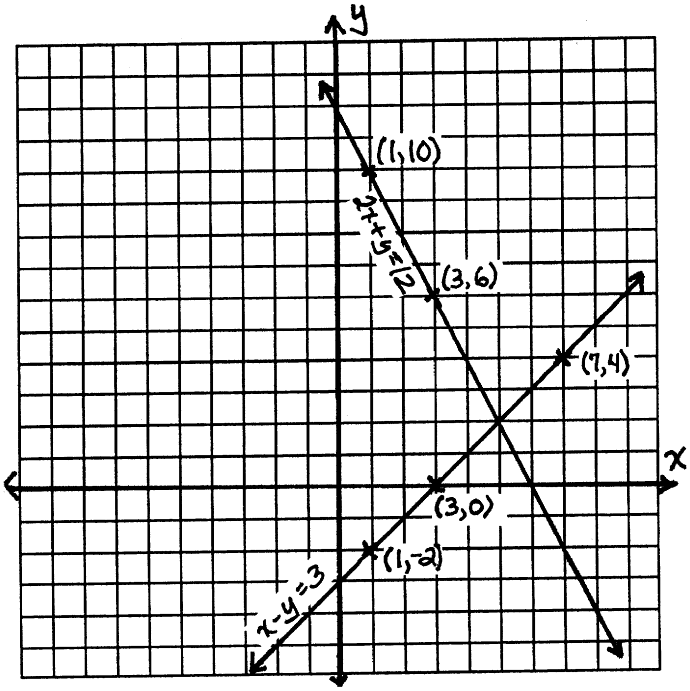
Correct lines
No credit is given for the solution or the check. The listing of a variety of coordinates does not identify the solution. (–2)
The checking of random points does not indicate the solution of the system. (–2) [GRAPH–3(b),3(c)]

$y = x - 3$

x	y
1	-2
3	0
7	4

$y = -2x + 12$

x	y
1	10
3	6
5	2



Check

$x - y = 3$
$7 - 4 = 3$
$3 = 3$ ✓

$3 - 0 = 3$
$3 = 3$ ✓

$2x + y = 12$
$2(3) + 6 = 12$
$6 + 6 = 12$ ✓

$2(5) + 2 = 12$
$10 + 2 = 12$ ✓

40. Question

Ramos buys some pens and pencils. He buys seven more pens than pencils. Pens cost \$0.45 each and pencils cost \$0.40 each. If he has \$10 to spend, what is the greatest number of each he can buy? [*Show or explain the procedure used to obtain your answer.*] [10]

Key

8 pencils
15 pens

Student Responses

(i) I guessed 8 pencils and 15 pens.

8 x .40 = 3.20
15 x .45 = 6.75
Answer: \$9.95

(+3)

(ii)

Pencils	Pens	Cost
5	5+7=12	\$2.00+5.40 = 7.40
6	6+7=13	8.25
7	14	9.10
8	15	9.95
9	16	10.80

The greatest number he can buy is 8 pencils and 15 pens.

(+10)

Rating Explanations

(i) The trial-and-error method should develop a pattern. Seven more pens than pencils should be mentioned, and a pattern developed which progresses toward the solution. If the student "guesses" the solution with the first guess, then a pair of numbers under and a pair of numbers over the solution should be checked to show that they do not work. [GEN–26] In addition, the student lists a response that does not answer the question.

(ii) All conditions for a complete solution have been met. [GEN-26]

41. Question

An 8- by 10-inch photo has a frame of uniform width placed around it.

a If the uniform width of the frame is x inches, express the outside dimensions of the picture frame in terms of x. [4]

b If the area of the picture and frame is 143 in^2, what is the uniform width of the frame? [6]

Key

a $8 + 2x$ and $10 + 2x$

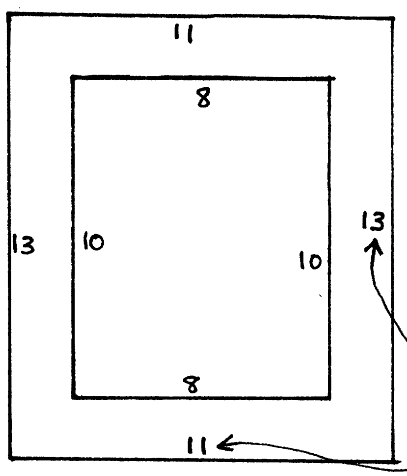
b $1\frac{1}{2}$ inches

Student Responses

(i)

(+4) a $8 + 2x$ by $10 + 2x$

(+5) b



$A = b \cdot h$

$A = 8 \cdot 10$

$A = 80$

$143 = 11 \cdot 13$

So... $8 + 2x = 11$
$10 + 2x = 13$

$8 + 2x = 11$
$\frac{2x}{2} = \frac{3}{2}$
$x = 1\frac{1}{2}$

$10 + 2x = 13$
$\frac{2x}{2} = \frac{3}{2}$
$x = 1\frac{1}{2}$

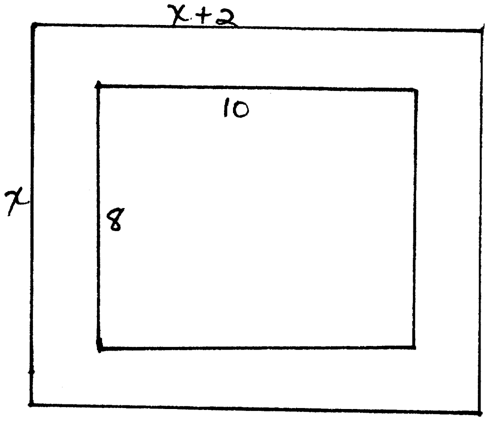
b $1\frac{1}{2}$ inches

Rating Explanations

a Correct representation

b The student recognized that the area of the picture and frame had to be the product of two numbers whose difference was 2. However, the student did not show that the other factors of 143 (1 and 143) do not produce a uniform width. (–1) [GEN-26]

(ii)



a) x and $x+2$ (+0) *a* Representation of the dimensions in terms of x is incorrect. (−4)

b) $x(x+2) = 143$

$x^2 + 2x = 143$ (+6) *b* This is an atypical solution that answers the question. [GEN–22]

$x^2 + 2x - 143 = 0$

$(x+13)(x-11) = 0$

$x + 13 = 0$ $\quad$ $x - 11 = 0$

$x = -13$ $\quad$ $x = 11$

Reject

width of frame $= \frac{11-8}{2} = \frac{3}{2} = 1.5$ in.

42. Question | **Key**

Use any method (algebraic, trial and error, making a table, etc.) to solve this problem. A written explanation of how you arrived at your answer is also acceptable. Show all work.

There are two pairs of integers that satisfy both of these conditions:

The larger integer is 9 more than the smaller integer.

The sum of the squares of the integers is 41.

a Find the two pairs of integers. [8]

Key: −5,4 and −4,5

b Show that one pair of integers found in part *a* satisfies both given conditions. [2]

Student Response

a To get the answer to this problem, I used the trial + error method. I tried ten different positive integers and none of them came out correctly. So I decided to try a negative number. The first number I chose was -5. I squared it to 25. Then I added 9 to -5 and came up with 4. I then squared 4 and came up with 16. Then I added the two together and came up with 41. Then I made the 4 negative and the 5 positive, squared them and came up with 41 again.
Answers: -5,4 and -4,5 (+8)

b
$$(-5)^2 + (-5+9)^2 = 41$$
$$25 + 4^2 = 41$$
$$25 + 16 = 41$$
$$41 = 41$$
(+2)

Rating Explanation

a The written explanation is acceptable. [GEN-26]

b Incorporates both conditions in one check

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